math

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National

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SAT

Notes .....

Practice test .....

past papers .....

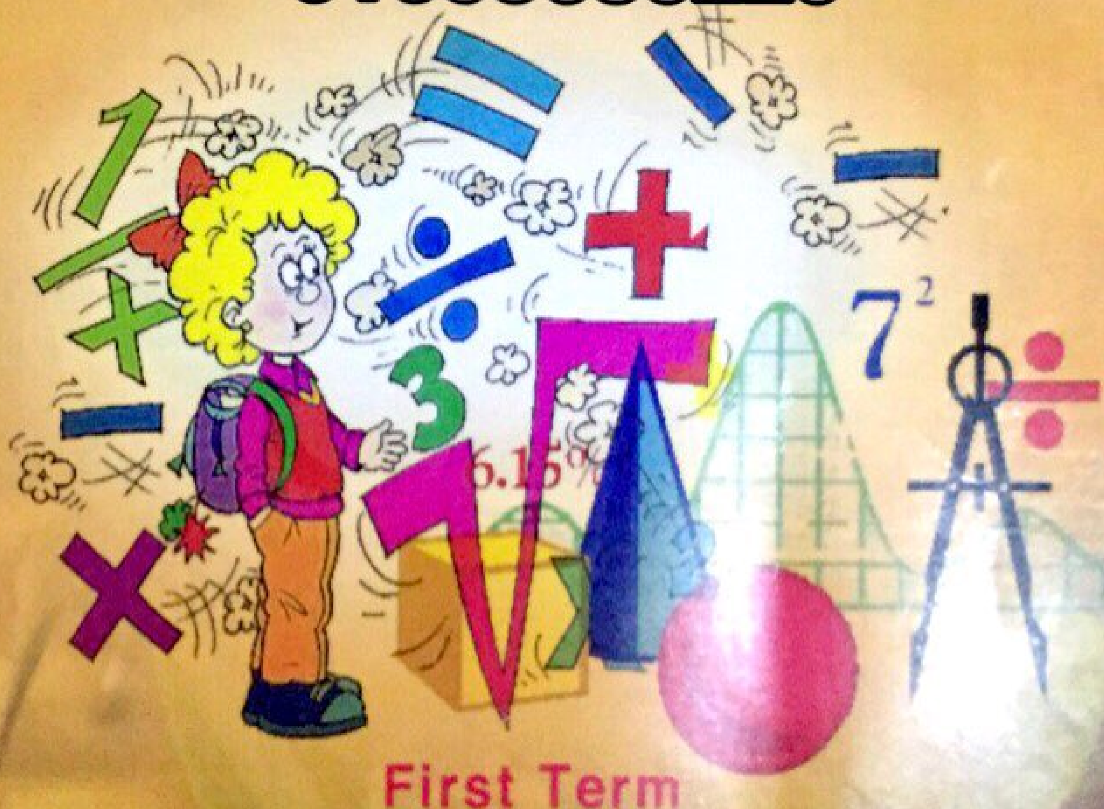
High Score is our Target

Geometry

Middle 2

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First Term

Unit Four

Medians of triangle

Isoceles Triangle

Lesson 1: Medians of triangle.

Lesson 2: Medians of triangle (follow)

Lesson 3: The isosceles triangle

Lesson 4: The converse of the isosceles triangle theorem

Lesson 5: Corollaries of the isosceles triangle theorem

Median of triangles

Definition:

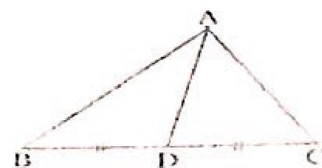
The median of a triangle is the line segment from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure:

If D is the midpoint of $\overline{BC}$, then

$\overline{AD}$ is a median of Δ ABC



Theorem 1:

The medians of a triangle are concurrent.

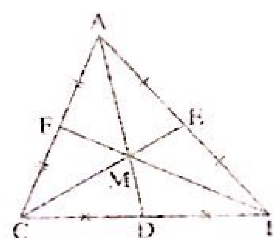
For example:

In the opposite figure:

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of Δ ABC,

And they are concurrent at M

(i.e. $\overline{AD} \cap \overline{BF} \cap \overline{CE}$ = {M})



Theorem 2:

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

For example:

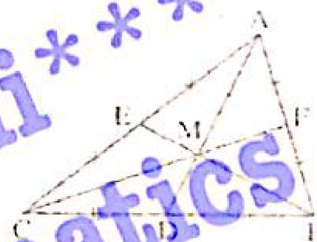
In the opposite figure:

M is the point of concurrence of the medians

Of Δ ABC, then:

1- MD = $\frac{1}{2}$ AM If AM = 6 cm, then MD = 3 cm.

2- CM = 2 FM If FM = 4 cm, then CM = 8 cm.



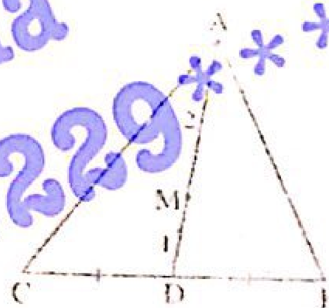
Fact:

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

In the opposite figure:

If $\overline{AD}$ is a median in Δ ABC and M $\in$ $\overline{AD}$ such that AM = 2 MD,

Then M is the point of intersection of the medians of Δ ABC



First Term-Geometry-Middle (2)

Exercise (1):

1- Complete the following:

1- In Δ ABC, if D is the midpoint of BC, then AD is called ....

2- The number of medians of the triangle is ....

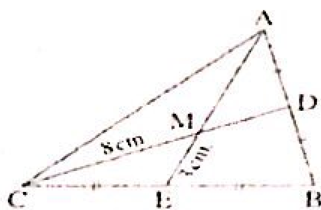
3- The medians of the triangle intersect at ....

4- The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... Fr its base.

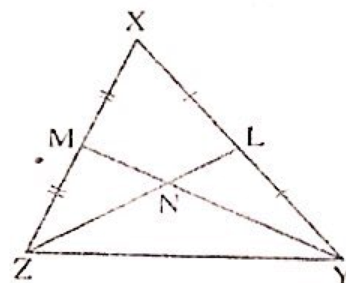
5- The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... Fr the vertex.

6- The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... Fr the base.

2- Using data given for each of the following figures, find the required below each figure:



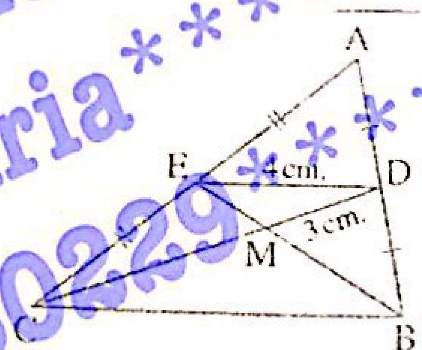
MA = ...cm,
MD = ...cm,
ME = ...cm,
And MC = ... CD



If LZ = 15 cm, YM = 18 cm
and XY = 20 cm,
then NL = ... cm.
NY = ... cm. and the perimeter of Δ NLY = ... cm.

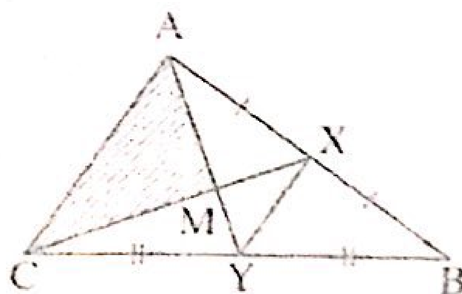
3- In the opposite figure:

If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$
And $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm, DM = 3 cm and BE = 6cm.
Find the perimeter of Δ BMC



First Term-Geometry-Middle (2)

1- In the opposite figure:

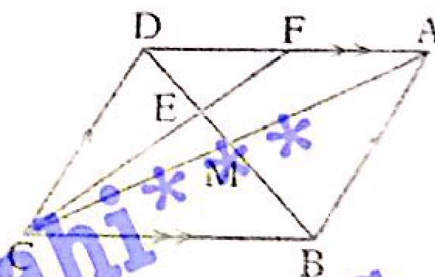


ABC is a triangle, X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{BC}$, XY = 5 cm
And $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm, YM = 3cm.
Find:

1- The perimeter of Δ MXY
2- The perimeter of Δ MAC

2- In the opposite figure:

ABCD is a parallelogram, its diagonals intersect at M,
E ∈ $\overline{DM}$ where DE = 2 EM, draw $\overrightarrow{CE}$ to cut $\overline{AD}$ at F
Prove that: AF = FD



3- ABC is a triangle where point D is the midpoint of BC and point M ∈ AD, AM = 2 MD
Draw CM to intersect AB at point E if EC = 12 cm, then find the length of EM

Median of triangle (follow)

Theorem 3:

In the right-angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

Given ABC is a triangle in which m ($\angle$ ABC) = 90°, $\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof In the figure ABCE: $\because$ $\overline{AC}$ and $\overline{BE}$ bisect each other

$\therefore$ The figure ABCE is a parallelogram

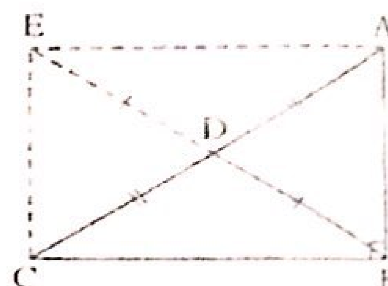
$\because$ m ($\angle$ ABC) = 90°

$\therefore$ The figure ABCE is a rectangle

$\therefore$ BE = AC

, $\because BD = \frac{1}{2} BE$

$\therefore BD = \frac{1}{2} AC$



The converse of theorem 3:

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given In Δ ABC, $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m ($\angle$ ABC) = 90°

Construction Draw $\overrightarrow{BD}$, then take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof $\because BD = \frac{1}{2} BE = \frac{1}{2} AC$ $\therefore$ BE = AC

$\therefore$ In the figure ABCE :

$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

$\therefore$ The figure ABCE is a rectangle

$\therefore$ m ($\angle$ ABC) = 90°



Corollary:

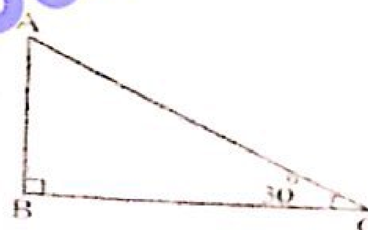
The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure:

If Δ ABC is right-angled at B and

m ($\angle$ C) = 30°, then $AB = \frac{1}{2} AC$



Exercise (2):

1- Complete the following:

1- In Δ ABC if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called.................

2- The medians of the triangle intersect at ....

3- The point of intersection of the medians of the triangle divides each of them in the ratio of .... :.. From the base.

4- The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point o ....

5- The length of the median of the right-angled triangle which is drawn from the vertex of the righ angle equals ....

6- If the length of the median of the triangle which is drawn from one of its vertices equals half th length of the opposite side to this vertex, then ....

7- The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....

8- The two base angles of the isosceles triangle are ....

9- If the angles of any triangle are equal in measure, then ....

10- If the measure of an angle in the isosceles triangle is 60°, then the triangle is ....

11- The axis of symmetry of the isosceles triangle is ....

12- The perpendicular projected from the vertex of the isosceles triangle to the base bisects ....

13- The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base i ....

14- If XYZ is a right-angled triangle at Y and XY = YZ, then m(∠X) = ...°

15- ABC is an isosceles triangle where AB = AC and m(∠A) = 110°, then m(∠B) = ...°

2- Complete the following:

1- In the opposite figure:

If M is the point of intersection of the medians of Δ ABC, then

(a) BD = ... BC

(b) AM = ... MD

(c) AM = ... AD

2- In each of the following figures, M is the point of intersection of the medians of the give 1

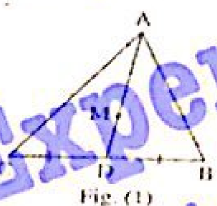


Fig. (1)



Fig. (2)

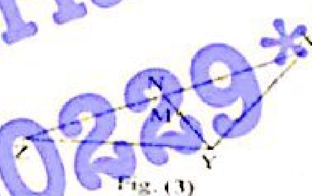


Fig. (3)

(a) In fig. (1): If AM = 2cm, then MD = ...cm

(b) In fig. (2): If MF = 1.5 cm, then DF = ...cm

(c) In fig. (3): If YN = 6 cm, then YM = ...cm

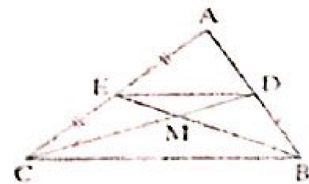
First Term-Geometry-Middle (2)

1- In the opposite figure:

(a) If DE = 3 cm, then BC = ... cm

(b) If CD = 4.5 cm, then CM = ... cm

(c) If ME = 1.2 cm, then BE = ... cm



2- In each of the following figures:

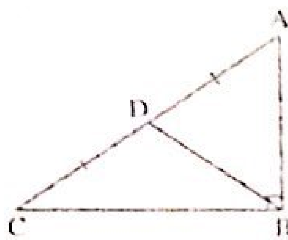


Fig. (1)

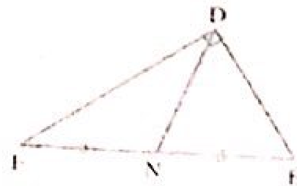


Fig. (2)

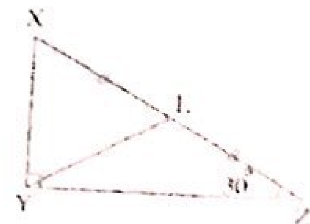


Fig. (3)

(a) In fig.(1): If AC = 8cm, then BD = ...cm

(b) In fig.(2): If DN = 3cm, then EN = ...cm

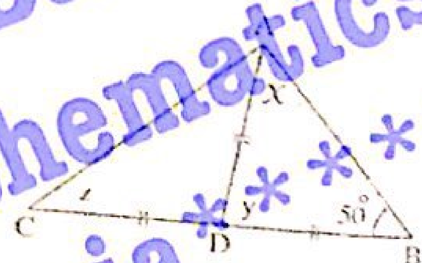
(c) In fig.(3): If XY = 3.5cm , then YL = ...cm

3- <u>In the opposite figure:</u>

(a) X = ...°

(b) Y = ...°

(c) Z = ...°



1- Using data registered in each figure:

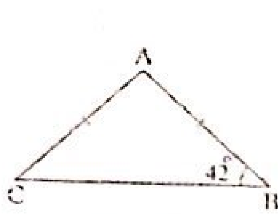


Fig. (1)

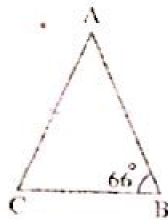


Fig. (2)

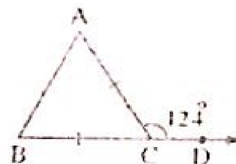


Fig. (3)

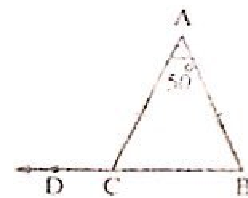
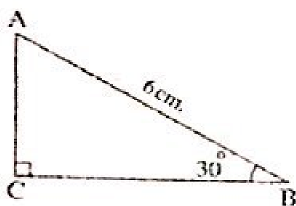
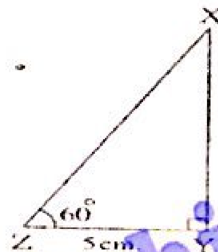


Fig. (4)

(a) Fig.(1) : m (∠ C) = ...
(b) Fig.(2) : m (∠ A) = ...
(c) Fig.(3) : m (∠ B) = ...
(d) Fig.(4) : m (∠ ACD) = ...

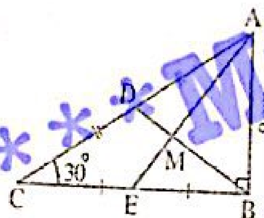
2- <u>Using data given for each of the following figures, find the required below eacl figure:</u>

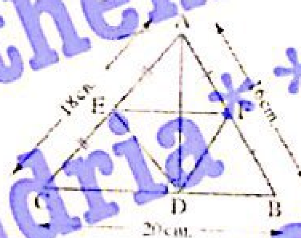




Ac = ...cm

XZ = ...cm





C = ...cm
D = ...cm
iD = ...BD
nd MD = ...cm

DF = ...cm
DE = ...cm
FE = ...cm
and the perimeter of Δ DEF = ...cm

First Term-Geometry-Middle (2)

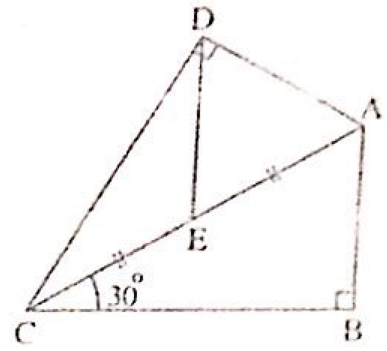
1- In the opposite figure:

M(∠ ABC) = m (∠ ADC) = 90°,

M(∠ ACB) = 30° and

E is the midpoint of $\overline{AC}$

Prove that: AB = DE



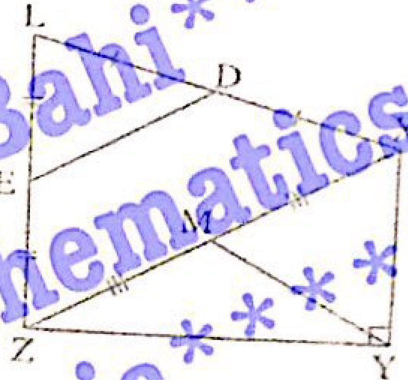
2- In the opposite figure:

m (∠ XYZ) = 90°, D is the midpoint of $\overline{XL}$

E is the midpoint of $\overline{ZL}$ and

M is the midpoint of $\overline{XZ}$

Prove that: DE = YM



First Term-Geometry-Middle (2)

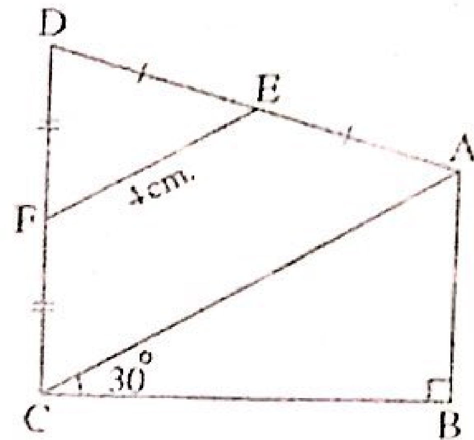
1- In the opposite figure:

ABCD is a quadrilateral in which m(∠ B) = 90°,

E is the midpoint of $\overline{AD}$ F is the midpoint of $\overline{CD}$,

M(∠ ACB) = 30° and EF = 4cm.

Find by proof the length of $\overline{AB}$



2- In the opposite figure:

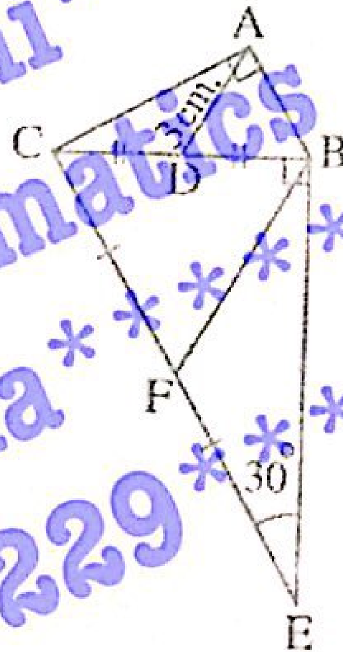
m (∠ BAC) = m (∠ CBE) = 90°,

m (∠ BEC) = 30°

D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively

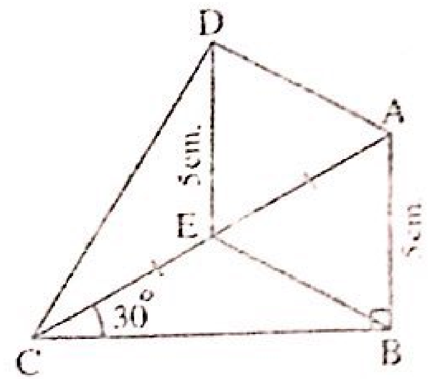
And AD = 3cm

Find the length of $\overline{BF}$



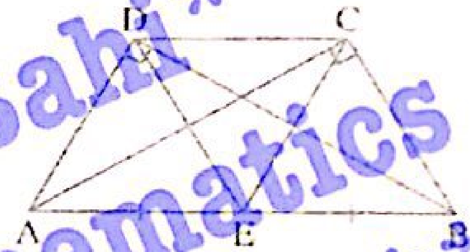
1- In the opposite figure:

ABC is a right-angled triangle at B,
m (∠ ACB) = 30°, AB = 5cm and
E is the midpoint of $\overline{AC}$, If DE = 5cm,
Prove that: m (∠ ADC) = 90°



2- In the opposite figure:

ADB is a right-angled triangle at D,
ACB is a right-angled triangle at C and
E is the midpoint of $\overline{AB}$
Prove that: Δ CED is an isosceles triangle.



First Term-Geometry-Middle (2)

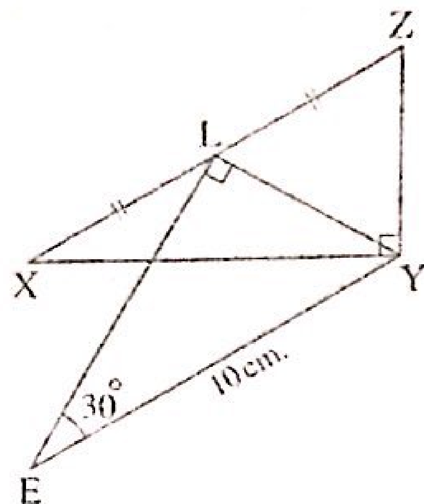
1- **In the opposite figure:**

m (∠ YLE) = 90°, m (∠ E) = 30° , YE = 10cm,

m (∠ XYZ) = 90° and

L is the midpoint of $\overline{XZ}$

Find by proof the length of $\overline{XZ}$



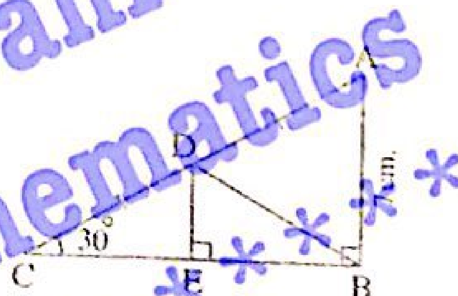
2- **In the opposite figure:**

ABC is a right-angled triangle at B,

D is the midpoint of $\overline{AC}$, $\overline{DE} \perp \overline{BC}$, AB = 7cm,

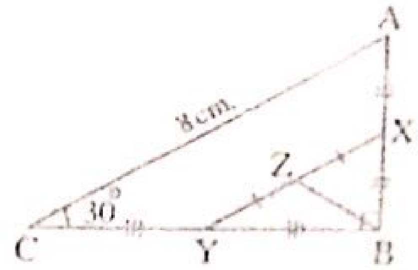
And m(∠ C) = 30°

Find the length of each of: $\overline{BD}$ and $\overline{DE}$



First Term-Geometry-Middle (2)

1- In the opposite figure:



ABC is a triangle in which m(∠ ABC) = 90°,

m(∠ C) = 30°,

X,Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$ respectively

And AC = 8cm

Find the length of each of: $\overline{AB}$, $\overline{XY}$ and $\overline{BZ}$

2- In the opposite figure:

ABCD is a square, E ∈ $\overline{BC}$ where m(∠ BAE) = 30° and

$\overline{DF} \perp \overline{AE}$ If AF = 4cm,

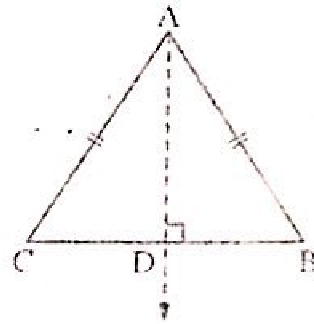
Calculate the area of the square ABCD



The isosceles triangle

Theorem 1:

The base angles of the isosceles triangle are congruent.



Given: ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P.: $\angle B \equiv \angle C$

Construction: Draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

Proof: $\because$ $\Delta\Delta$ ABD, ADC in which:

M($\angle$ ADB) = m($\angle$ ADC) = 90° (const)

$\overline{AB} \equiv \overline{AC}$ (given)

$\overline{AD}$ is a common side

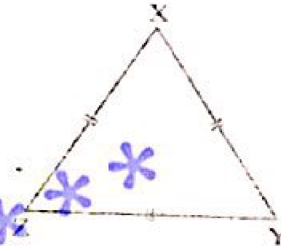
$\therefore$ Δ ADB $\equiv$ Δ ADC, then

We deduce that $\angle B \equiv \angle C$

Corollary:

If the triangle is equilateral, then it is equiangular where each angle measure is 60°

For example:



In the opposite figure:

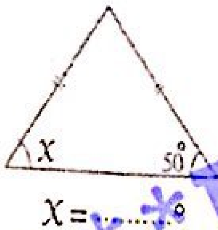
If XYZ is a triangle in which

XY = YZ = ZX,

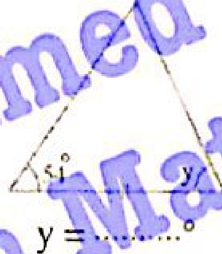
Then m ($\angle$ X) = m ($\angle$ Y) = m ($\angle$ Z) = 60°

Exercise (3):

- In each of the following, find the value of the symbol used for the measure of the angle:



X = ...°



y = ...°



ℓ = ...° ,

z = ...°

- Complete the following:

1- In Δ DEF, if DE = DF, then m($\angle$ E) = m($\angle$......).

2- In the isosceles triangle, if the measure of one of the two base angles is 65°, then the measure o its vertex angle equals ....°

3- In the isosceles triangle, if the measure of the vertex angle = 40°, then the measure of one of th two base angles equals ....°

4- In Δ ABC, if AB = AC and m($\angle$ A) = 80°, then m($\angle$ B) = m($\angle$....) = ...°

1- Choose the correct answer from those given:

2- The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

3- If Δ ABC is right-angled at A and AB = AC, then m(??? B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

3- If the measure of one of the two base angles in the isosceles triangle = 30°, then the triangle is ....

(a) Obtuse-angled (b) acute-angled

(c) right-angled (d) equilateral triangle

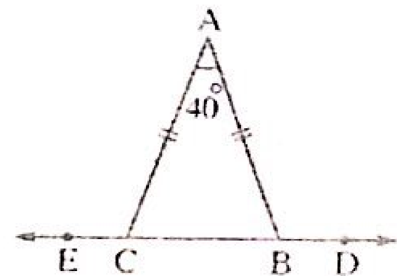
2- In the opposite figure:

ABC is an isosceles triangle in which AB = AC,

M(∠ A) = 40° and D ∈ $\overrightarrow{CB}$, E ∈ $\overrightarrow{CB}$

First: Find m(∠ ABC)

Second: Prove that: ∠ ABD ≡ ∠ ACE

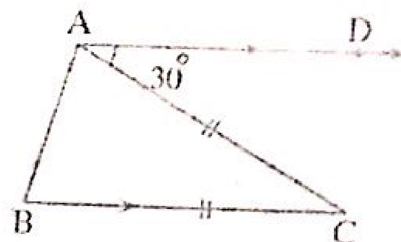


1- **In the opposite figure:**

ABC is a triangle in which AC = BC,

$\overrightarrow{AD}$ // $\overline{BC}$ and m(∠ DAC) = 30°

Find the measures of the angles of Δ ABC

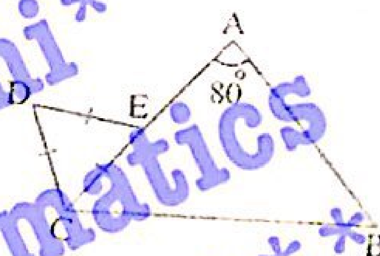


- **In the opposite figure:**

AB = AC, m (∠ BAC) = 80°

And CE = ED = CD

Find by proof: m (∠ BCD)

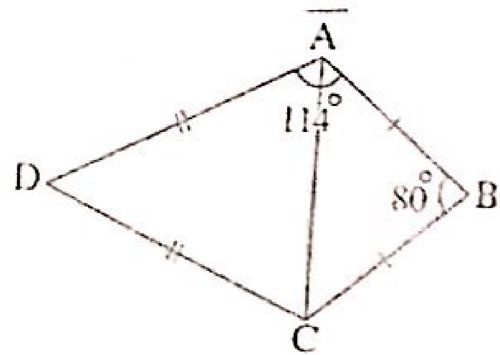


1- <u>In the opposite figure:</u>

AB = BC, AD = CD, m (∠ BAD) = 114°

And m (∠ B) = 80°

Find: m (∠ ADC)



2- <u>In the opposite figure:</u>

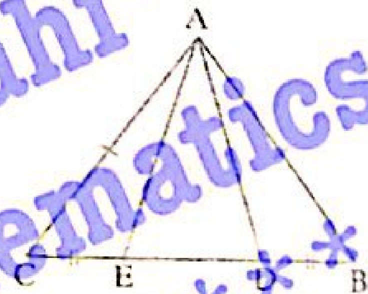
ABC is an isosceles triangle in which AB = AC, D ∈ $\overline{BC}$

And E ∈ $\overline{BC}$, such BD = EC

Prove that:

1- Δ ADE is an isosceles triangle.

2- ∠ AED ≡ ∠ ADE

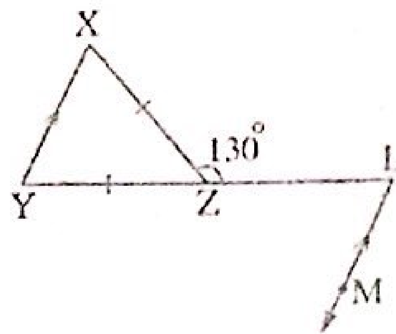


- In the opposite figure:

$Z \in \overline{LY}$, XZ = YZ, m(∠ LZX) = 130°

And $\overrightarrow{LM} // \overline{XY}$

Find: m (∠ MLY)



- In the opposite figure:

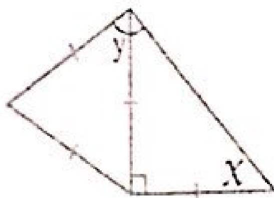
AB = AC, m (∠ B) = 2x + 13°

and m(∠ C) = 3x - 17°

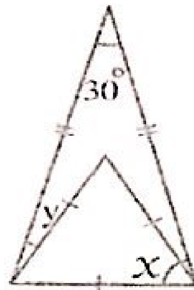
Find the measures of the angles of Δ ABC



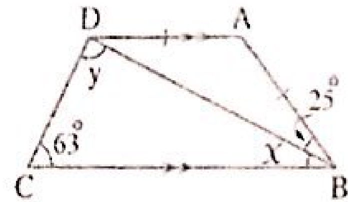
1- <u>In each of the following figures, find the value of the symbol used for the measure of the angle:</u>



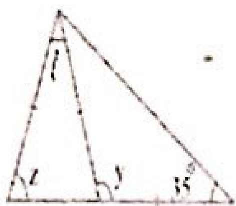
$x =$ ....° ,
y = ...°



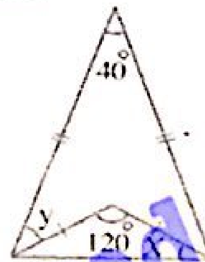
$x =$ ....° ,
y = ...°



$x =$ ....° ,
y = ...°



y = ...° , l = ...° ,
z = ...°



$x =$ ....° ,
y = ...°

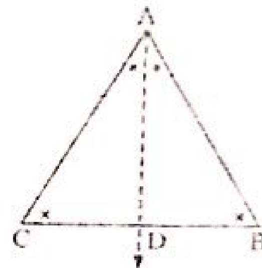


$x =$ ....°

The converse of the isosceles triangle theorem

Theorem 2:

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.



Given: ABC is a triangle in which $\angle B \equiv \angle C$

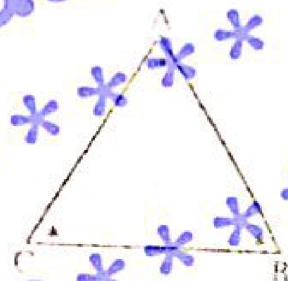
R.T.P.: $\overline{AB} \equiv \overline{AC}$

Construction: Bisect $\angle$ BAC by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof:
$\because \angle B \equiv \angle C$
$\therefore m(\angle B) = m(\angle C)$
$\because \overrightarrow{AD}$ bisects $\angle BAC$
$\therefore m(\angle BAD) = m(\angle CAD)$
$\because$ The sum of measures of the interior angles of the triangle = 180°
$\therefore m(\angle ADB) = m(\angle ADC)$
$\therefore$ In $\Delta\Delta$ ABD and ACD:
$\overline{AD}$ is a common side
$M(\angle BAD) = m(\angle CAD)$ (const)
$M(\angle ADB) = m(\angle ADC)$ (by proof)
$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that
$\overline{AB} \equiv \overline{AC}$, then Δ ABC is an isosceles triangle.

Corollary:

If the angles of a triangle are congruent, then the triangle is equilateral.



For example:

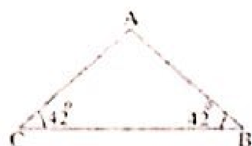
If ΔABC is a triangle in which:
$\angle A \equiv \angle B \equiv \angle C$, then AB = BC = CA
i.e. Δ ABC is an equilateral triangle.

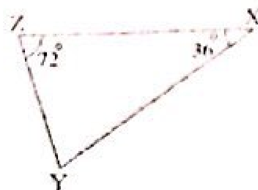
Remark:

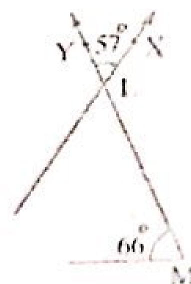
The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

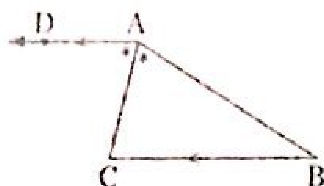
Exercise (4):

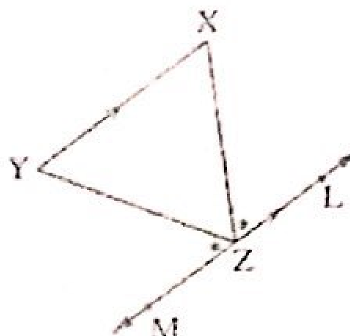
1- In each of the following figures, write the equal sides in length:











2- Complete the following:

- If the three angles in the triangle are congruent, then the triangle is ....
- In Δ ABC, if m (∠ A) = 50° and m (∠ B) = 80°, then the triangle is ....
- If the measure of one angle in the right-angled triangle is 45°, then the triangle is ....
- If the measure of one angle of an isosceles triangle is 60°, then the triangle is ....
- ABC is a triangle in which AB = AC and m (∠A) = 60° If its perimeter = 18 cm, then BC = ... Cm.

3- ABC is a triangle in which D ∈ $\overline{AB}$ and E ∈ $\overline{BC}$ such that BD = BE
So if $\overline{DE}$ // $\overline{AC}$, Prove that AB = BC

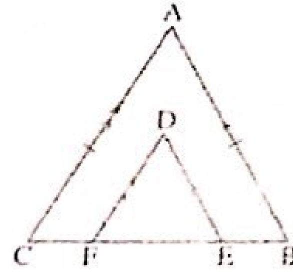
1- In the opposite figure:

AB = AC, if $\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

Prove that:

1- DE = DF

2- m (∠BAC) = m (∠EDF)

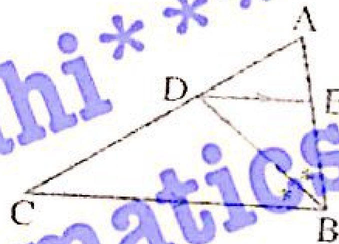


2- In the opposite figure:

ABC is a triangle,

$\overrightarrow{BD}$ bisects ∠ABC and $\overline{ED} \parallel \overline{BC}$ where E ∈ $\overline{AB}$

Prove that: Δ EBD is an isosceles triangle.

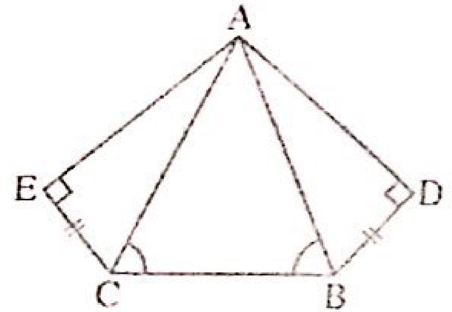


1- In the opposite figure:

BD = CE, m ($\angle$ ABC) = m ($\angle$ ACB)

AND m ($\angle$ D) = m ($\angle$ E) = 90°

Prove that: m ($\angle$ DAB) = m ($\angle$ CAE)

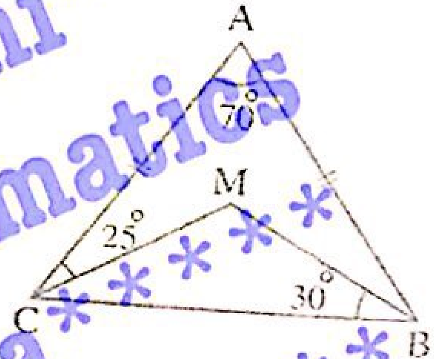


2- In the opposite figure:

ABC is a triangle in which AB = AC, m ($\angle$ A) = 70°

m ($\angle$ MCA) = 25° and m ($\angle$ MBC) = 30°

prove that: Δ MBC is an isosceles triangle.



1- ABC is a triangle in which AB = AC, $\overrightarrow{BD}$ bisects $\angle$ ABC and $\overrightarrow{CD}$ bisects $\angle$ ACB

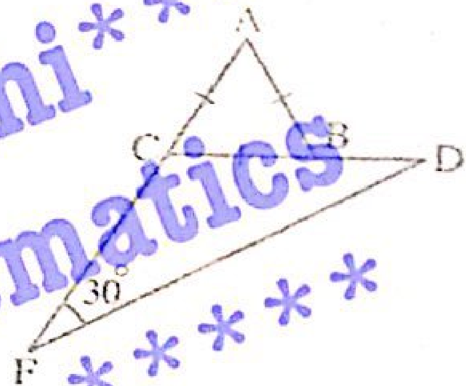
Prove that: Δ DBC is an isosceles triangle.

2- In the opposite figure:

ABC is an equilateral triangle, F $\in \overrightarrow{AC}$,

D $\in \overrightarrow{CB}$ and m ($\angle$ DFC) = 30°

Prove that: Δ DCF is an isosceles triangle.



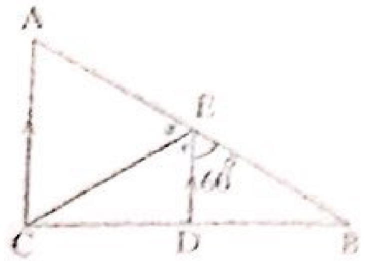
1- In the opposite figure:

ABC is a triangle in which E $\in \overline{AB}$,

$\overline{ED} \,//\, \overline{AC}$, m ($\angle$ BED) = 60°

And $\overrightarrow{EC}$ bisects $\angle$ AED

Prove that: Δ AEC is an equilateral triangle.



2- In the opposite figure:

$\angle$ ADE $\equiv \angle$ AED,

B, D, E, C are collinear and BD = CE

Prove that: Δ ABC is an isosceles triangle.

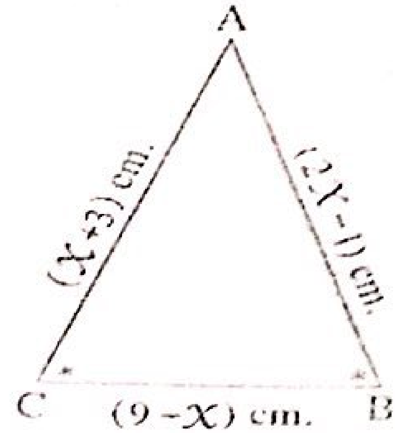


1- **In the opposite figure:**

ABC is a triangle in which:

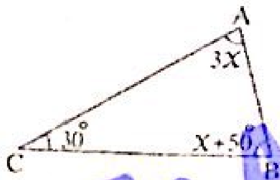
m (∠ B) = m (∠ C)

Find the perimeter of the triangle.

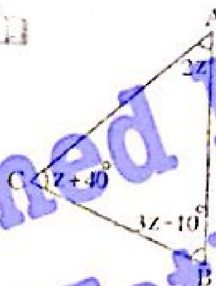


2- **In each of the following figures, write the equal sides in length showing the steps o solution:**

(1)



(2)



(3)



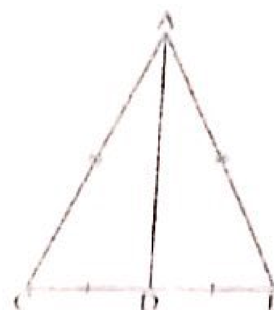
Corollaries of the isosceles triangle theorem

Corollary 1:

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base

In the opposite figure:
ABC is a triangle in which AB = AC and
$\overline{AD}$ is a median, then:

1- $\overrightarrow{AD}$ bisects $\angle$ BAC
   i.e. m ($\angle$ BAD) = m ($\angle$ CAD)
2- $\overline{AD} \perp \overline{BC}$

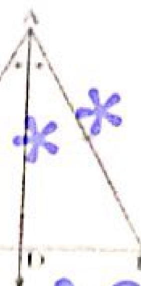


Corollary 2:

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure:
ABC is a triangle in which AB = AC and
$\overrightarrow{AD}$ bisects $\angle$ BAC, then:

1- D is the midpoint of $\overline{BC}$
   i.e. BD = CD
2- $\overline{AD} \perp \overline{BC}$



Corollary 3:

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure:
ABC is a triangle in which AB = AC and
$\overleftrightarrow{AD} \perp \overline{BC}$ then:

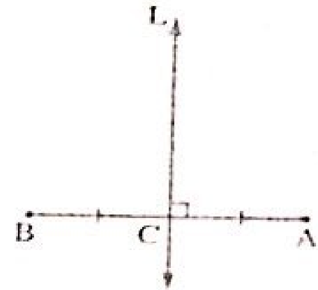
1- D is the midpoint of $\overline{BC}$
   i.e. BD = CD
2- m ($\angle$ BAD) = m ($\angle$ CAD)



Axis of symmetry of a line segment

Definition:

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that lin segment, in brief it is known as the axis of a line segment.



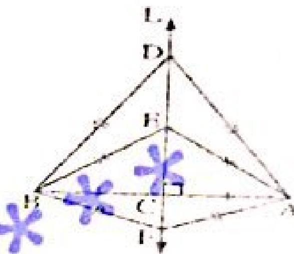
In the opposite figure:

If the straight line L $\perp \overline{AB}$ and C $\in$ the straight Line L where C is the midpoint of $\overline{AB}$, then

The straight line L is called the axis of $\overline{AB}$

roperty:

ny point on the axis of symmetry of a line segment is at equal distance from its terminals (end point).



n the opposite figure:

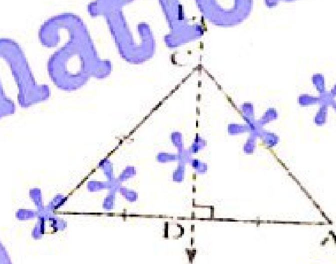
f the straight line L is the axis of $\overline{AB}$,

$\in$ L, E $\in$ L and F $\in$ L, then

A = DB, EA = EB and FA = FB

he converse of the pervious property is true:

e. If a point is at equal distances from the two terminals of a line segment, then this point lies on the axi f this line segment.



n the opposite figure:

f C is a point such hat CA = CB, then he point C lies on the axis of $\overline{AB}$,

xis of symmetry of the isosceles triangle

he isosceles has one axis of symmetry.

t is the straight line drawn from the vertex angle perpendicular to i



or example:

f ABC is an isosceles triangle where B = AC and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of he isosceles triangle ABC

General Exercises:

First: Completion questions:

1- Complete the following:

1- In Δ ABC if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called................

2- The medians of the triangle intersect at ....

3- The point of intersection of the medians of the triangle divides each of them in the ratio of .... : From the base.

4- The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of .....

5- The length of the median of the right-angled triangle which is drawn from the vertex of the righ angle equals ....

6- If the length of the median of the triangle which is drawn from one of its vertices equals half th length of the opposite side to this vertex, then ....

7- The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....

8- The two base angles of the isosceles triangle are ....

9- If the angles of any triangle are equal in measure, then ....

10- If the measure of an angle in the isosceles triangle is 60°, then the triangle is ....

11- The axis of symmetry of the isosceles triangle is ....

12- The perpendicular projected from the vertex of the isosceles triangle to the base bisects ....

13- The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base i ....

14- If XYZ is a right-angled triangle at Y and XY = YZ, then m(∠X) = ...°

15- ABC is an isosceles triangle where AB = AC and m(∠A) = 110°, then m(∠B) = ...°

2- Complete the following:

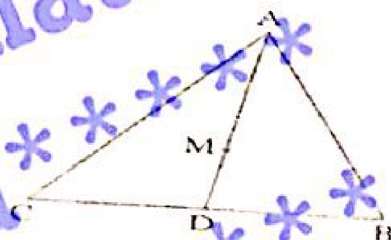
1- In the opposite figure:

If M is the point of intersection of the medians of Δ ABC, then

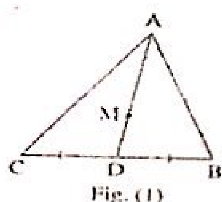
(a) BD = ... BC

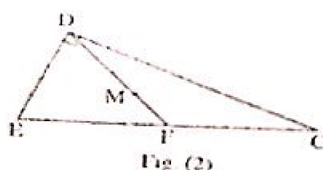
(b) AM = ... MD

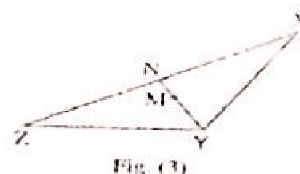
(c) AM = ... AD



1- In each of the following figures, M is the point of intersection of the medians of the given triangle:







(a) In fig. (1): If AM = 2cm, then MD = ...cm

(b) In fig. (2): If MF = 1.5 cm, then DF = ...cm

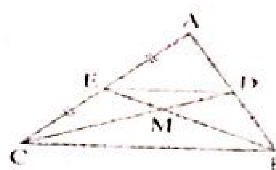
(c) In fig. (3): If YN = 6 cm, then YM = ...cm

2- In the opposite figure:

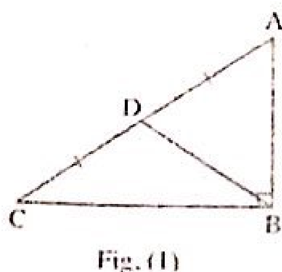
(a) If DE = 3 cm, then BC = ... cm

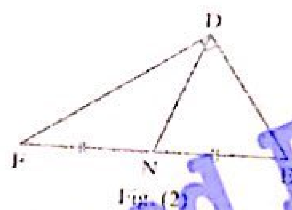
(b) If CD = 4.5 cm, then CM = ... cm

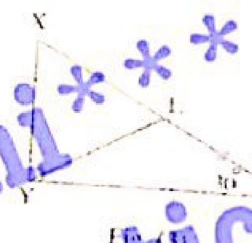
(c) If ME = 1.2 cm, then BE = ... cm



3- In each of the following figures:







(a) In fig.(1): If AC = 8cm, then BD = ...cm

(b) In fig.(2): If DN = 3cm, then EN = ...cm

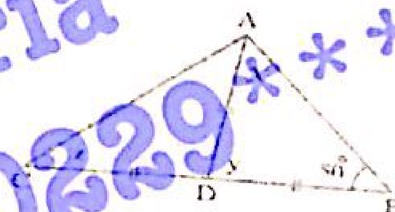
(c) In fig.(3): If XY = 3.5cm , then YL = ...cm

4- <u>In the opposite figure:</u>

(a) X = ...°

(b) Y = ...°

(c) Z = ...°



First Term-Geometry-Middle (2)

1- Using data registered in each figure:

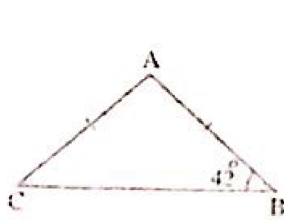


Fig.(1)

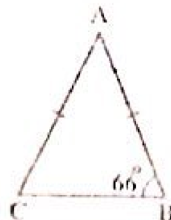


Fig.(2)

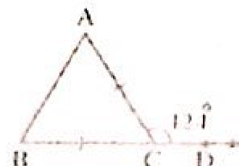


Fig.(3)

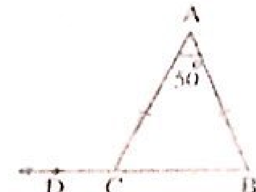


Fig.(4)

(a) Fig.(1) : m ($\angle$ C) = ...

(b) Fig.(2) : m ($\angle$ A) = ...

(c) Fig.(3) : m ($\angle$ B) = ...

(d) Fig.(4) : m ($\angle$ ACD) = ...

Second: Multiple choice questions:

Choose the correct answer from those given:

1- If M is the point of intersection of the medians of Δ ABC and D is the midpoint of $\overline{BC}$, then AD = ..

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

2- The point of intersection of the medians of the triangle divides each of them with the ratio ......:...... from the vertex.

(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 3 : 2

3- If M is the point of intersections of the medians of the triangle in Δ ABC and $\overline{AX}$ is a median o length 6 cm , then AM = ...

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

4- ABCD is a rectangle; M is the point of intersection of its diagonals. If the length of the diagonal is cm, then the length of the median $\overline{AM}$ equals ....

(a) 2 cm (b) 3 cm (c) 6 cm (d) 12 cm

5- The measure of the exterior angle of the equilateral triangle equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

6- If the measure of the vertex angle of the isosceles triangle equals 50°, then the measure of eac angle of its base equals ....

(a) 40° (b) 65° (c) 70° (d) 130°

7- If the measure of one of the two base angles of the isosceles triangle equals 40°, then the measur of the vertex angle is ....

(a) 40° (b) 50° (c) 80° (d) 100°

8- The axis of symmetry of line segment is the straight line which ....

(a) Is parallel to the line segment.

(b) Is perpendicular to the line segment.

(c) Bisects the line segment.

(d) Is the perpendicular bisector of the line segment.

9- If XA = XB and YA = YB, then $\overleftrightarrow{XY}$ .... $\overline{AB}$

(a) // (b) $\perp$ (c) = (d) $\equiv$

10- If A lies on the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$

(a) // (b) $\perp$ (c) = (d) $\equiv$

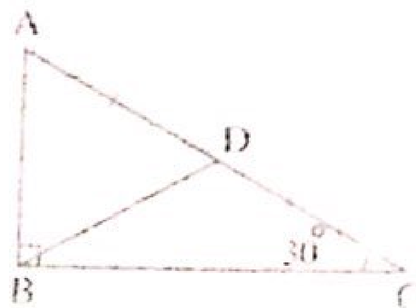
Third : Easy questions:

1- **In the opposite figure:**

M (∠ ABC) = 90°, D is the midpoint of $\overline{AC}$

, m (∠ C) = 30°

Prove that: Δ ABC is equilateral.

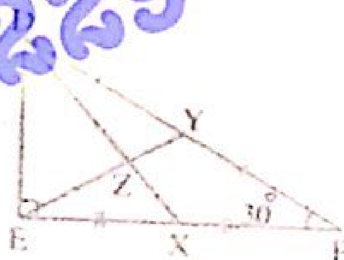


2- In the opposite figure:

m (∠ DEF) = 90°, X and Y are the midpoint of $\overline{EF}$, $\overline{DF}$ respectively, m (∠ F) = 30°,

DF = 12 cm, XZ = 2.5 cm.

Find the perimeter of Δ DEZ



1- In the opposite figure:

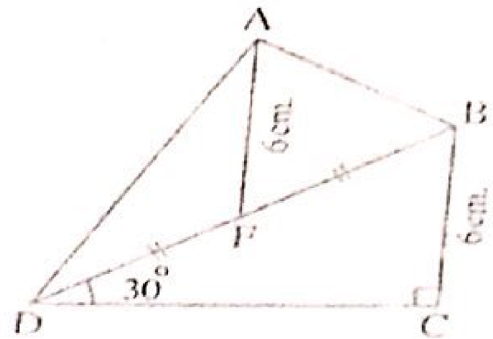
m (∠C) = 90°, $\overline{AF}$ is a median of Δ ABD

, m (∠BDC) = 30°

, BC = AF = 6cm.

1- Find the length of $\overline{BD}$

2- Prove that m (∠BAD) = 90°



2- In the opposite figure:

m (∠ABC) = 90°, m (∠ACB) = 30°,

Y and X are the midpoints of $\overline{CD}$ and $\overline{AD}$ respectively

Prove that: XY = AB

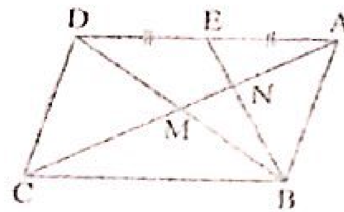
1- In the opposite figure:

ABCD is a parallelogram, its diagonals intersect at M

, E is the midpoint of $\overline{AD}$ and

$\overline{BE} \cap \overline{AC} = \{N\}$

Prove that: $AN = \frac{1}{3} AC$



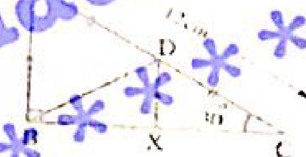
2- In the opposite figure:

m (∠ ABC) = 90°, m (∠ C) = 30°,

D is the midpoint of $\overline{AC}$, $\overline{DX} \parallel \overline{AB}$

AC = 12 cm

Find the length of each of: $\overline{BD}, \overline{BA}$ and $\overline{DX}$

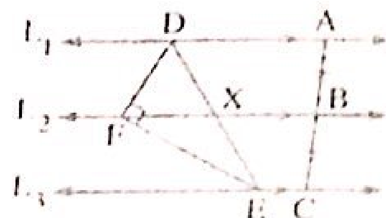


1- In the opposite figure:

$L_1 // L_2 // L_3$, AB = BC and

m ($\angle$DFE) = 90°,

prove that: $FX = \frac{1}{2} DE$

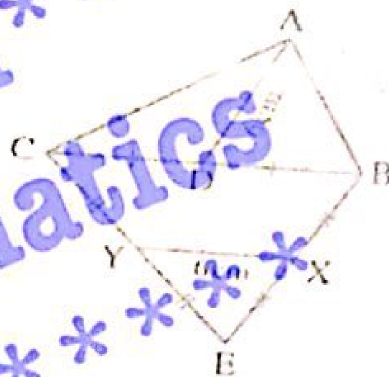


2- In the opposite figure:

$\overline{AD}$ is a median of Δ ABC, X and Y are the midpoints of $\overline{BE}$ and $\overline{CE}$ respectively,

AD = XY = 6 cm.

Prove that: m ($\angle$BAC) = 90°

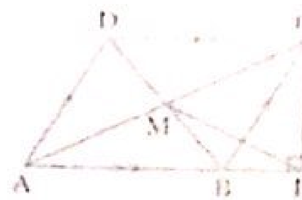


1- In the opposite figure:

ABCD is a parallelogram, M is the point of intersection Of its diagonals, $\overline{CE} \perp \overline{AB}$ such that $\overline{CE} \cap \overline{AB} = \{E\}$, m ($\angle$ DCA) = 30°

And Ac = 18 cm

Prove that: Δ CEM is equilateral and find its perimeter.



2- In the opposite figure:

m ($\angle$ BAC) = m ($\angle$ CBE) = 90°,

m ($\angle$ BEC) = 30°, D and F are the midpoints of $\overline{BC}$ *and* $\overline{CE}$ respectively

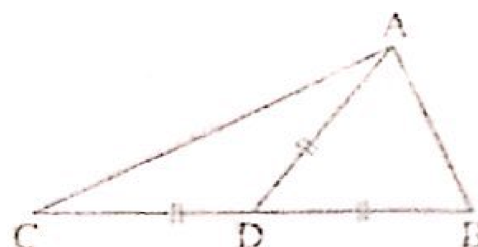
Prove that: AD = $\frac{1}{2}$ BF



1- In the opposite figure:

DA = DB = DC

Prove that: m (∠BAC) = 90°



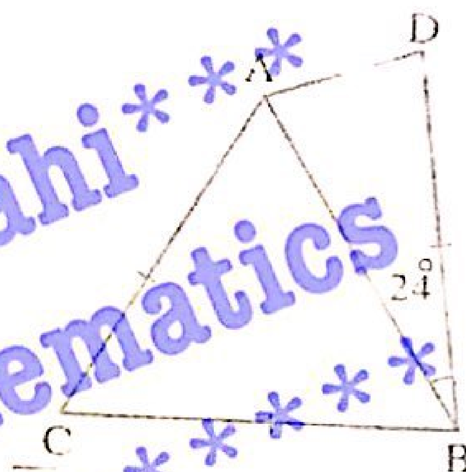
2- In the opposite figure:

ACBD is a quadrilateral in which

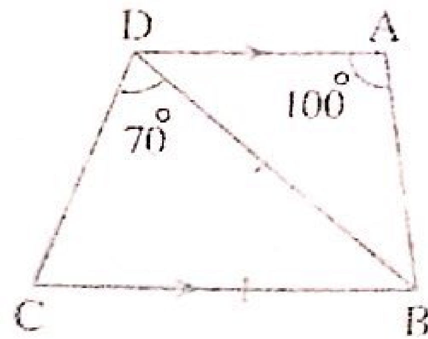
AB = BC = CA = BD

, m (∠ABD) = 24°,

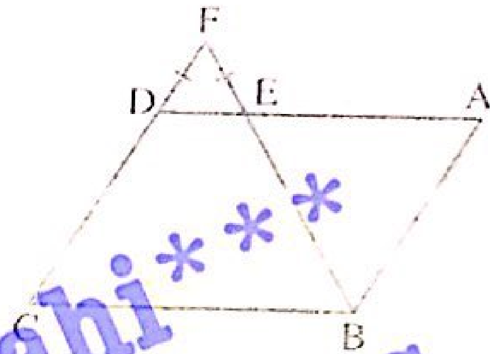
Find : m (∠CAD)



1- In the opposite figure:
$\overline{AD} \parallel \overline{BC}$, m ($\angle$ BAD) = 100°,
m ($\angle$ BDC) = 70°,
and BD = BC
prove that: Δ ABD is isosceles.



2- In the opposite figure:
ABCD is a parallelogram, E $\in \overline{AD}$
, $\overrightarrow{BE} \cap \overrightarrow{CD}$ = {F} such that EF = DF
Prove that: Δ BAE is isosceles.



3- In the opposite figure:
ABCD is a square and E is a point inside it such that
m ($\angle$ EAB) = m ($\angle$ EBA)
Prove that: Δ ECD is isosceles.



Comparing the lengths of sides in a triangle

Theorm:

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to side greater in length than that opposite to the other angle.

Given ABC is a triangle in which m ($\angle$C) 〉 m ($\angle$B)

R.T.P. AB 〉 AC



Proof $\because$ $\overline{AB}$ and $\overline{AC}$ are two line segments

$\therefore$ One of the following cases should be verified:

1- AB 〉 AC 2- AB = AC 3-AB 〈

Unless AB 〉 AC, then either AB = AC or AB 〈 AC

* If AB = AC, then m ($\angle$C) = m ($\angle$B) and this contradicts the given where m ($\angle$C) 〉 m ($\angle$B)

* If AB 〈 AC, then m ($\angle$C) 〈 m ($\angle$B) according to the previous theorem. Again this contradicts the given where m ($\angle$C) 〉 m ($\angle$B)

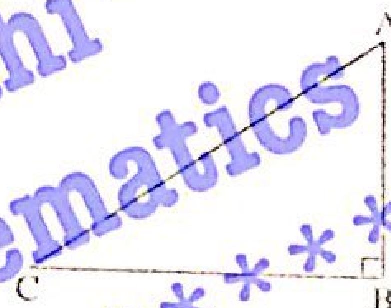
$\therefore$ It should be that AB 〉 AC

Corollaries:

Corollary 1:

In the right-angled triangle, the hypotenuse is the longest side.

In the opposite figure:



If Δ ABC is right-angled at B, then m ($\angle$B) 〉 m ($\angle$A).

m ($\angle$B) 〉 m ($\angle$C) because $\angle$B is a right angle and each of $\angle$A and $\angle$C is acute, so we find that:

AC 〉 BC and AC 〉 AB (according to the previous theorem).

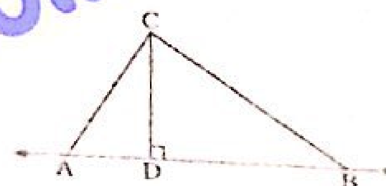
Notice that:

In the obtuse-angled triangle, the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2:

The length of the perpendicular line segment drawn from a point outside a straight line to this i shorter than any line segment drawn from this point to the given straight line.

In the opposite figure:



IF C $\notin \overleftrightarrow{AB}$ and D $\in \overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$,

Then $\overline{CB}$ is the hypotenuse in Δ CBD

Which is right-angled at D,

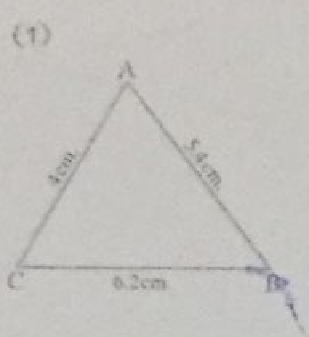
$\overline{CA}$ is the hypotenuse in Δ CDA which is right-angled at D so on...

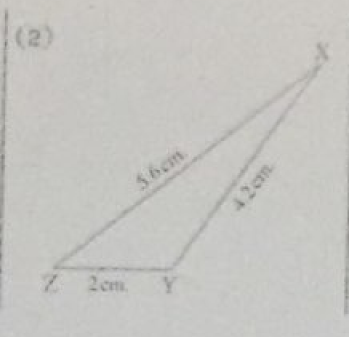
According to corollary 1, we find that CB 〉 CD, CA 〉 CD and so on...

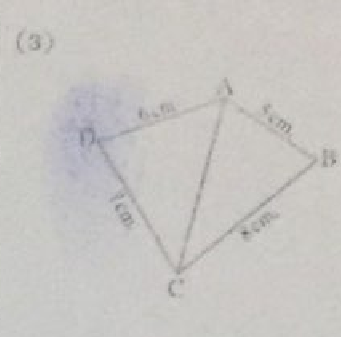
i.e. CD 〈 CB and CD 〈 CA

Exercise:

1- In each of the following figures, complete using ($>$ or $<$):







m (∠A)........ m (∠B)
m (∠A)........ m (∠C)
m (∠B)........ m (∠C)

m (∠Z)........ m (∠Y)
m (∠X)........ m (∠Y)
m (∠Z)........ m (∠X)

m (∠BAC)........ m (∠BCA)
m (∠DAC)........ m (∠DCA)
m (∠BAD)........ m (∠BCD)

2- Arrange the measures of the angles of Δ ABC in each of the following cases ascendingly:

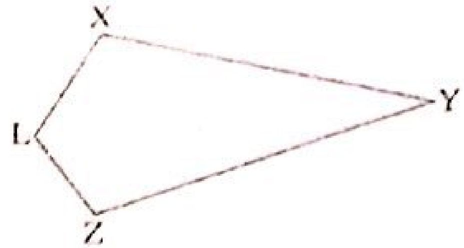
1- If AB = 12cm, BC = 15cm and AC = 10cm

2- If AB = 5.7 cm, BC = 8.5cm and AC = 6cm

1- In the opposite figure:

XY 〉 XL and YZ 〉 ZL

Prove that: m (∠XLZ) 〉 m (∠XYZ)



2- In the opposite figure:

ABCD is a quadrilateral in which

AD = DC and BC 〉 AB

Prove that: m (∠A) 〉 m (∠C)



1- ABCD is a quadrilateral in which: $\overline{AB}$ is the longest side, $\overline{CD}$ is the shortest one

Prove that: m (∠ BCD) 〉 m (∠ BAD)

2- **In the opposite figure:**

ABC is a triangle,

AB 〉 AC and $\overline{XY}$ // $\overline{BC}$

Prove that: m (∠ AYX) 〉 m (∠ AXY)

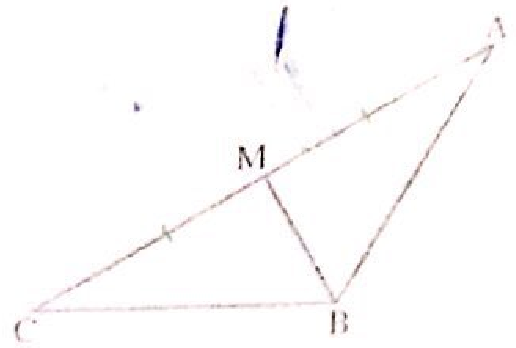


1- In the opposite figure:

$\overline{BM}$ is a median in the triangle ABC and

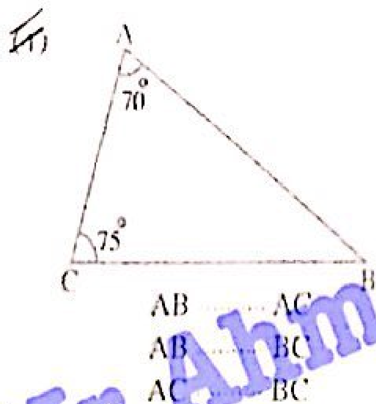
$BM < AM$

Prove that: $\angle ABC$ is an obtuse angle



2- In the following figures, complete using $>$, $<$ $or =$

(1)



AB ...... AC

AB ...... BC

AC ...... BC

(2)



XY ...... XZ

YZ ...... XY

YZ ...... XZ

(3)



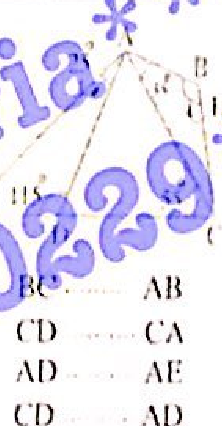
AC ...... BC

BC ...... DB

AC ...... BD

CA ...... DC

(4)



BC ...... AB

CD ...... CA

AD ...... AE

CD ...... AD

1- ABC is a triangle in which: m (∠ A) = 40° and m (∠ B) = 75°
Order the length of the sides of the triangle descendingly.

2- **In the opposite figure:**
ABC is a triangle, D ∈ $\overrightarrow{CB}$,
E ∈ $\overrightarrow{AC}$, m (∠ ABD) = 110° and
m (∠ BCE) = 120°
prove that: AB > BC

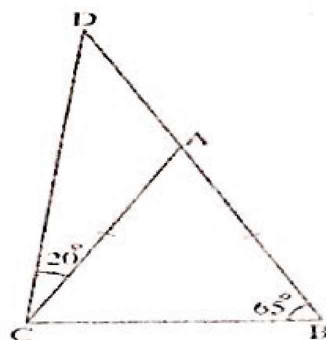


1- In the opposite figure:

AB = AC, m (∠ ABC) = 65°,

m (∠ ACD) = 20°, A ∈ $\overline{BD}$

prove that: AB 〉 AD



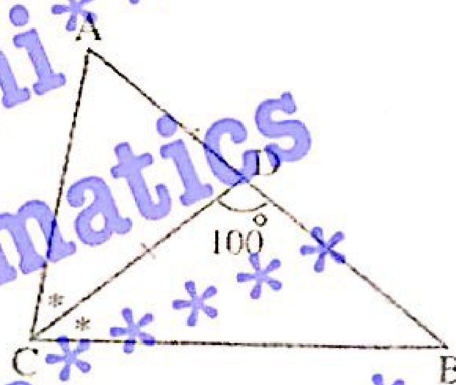
2- In the opposite figure:

ABC is a triangle, $\overrightarrow{CD}$ bisects ∠C

and intersects $\overline{AB}$ at point D,

m (∠ BDC) = 100° and DB = DC

prove that: AC 〉 DB

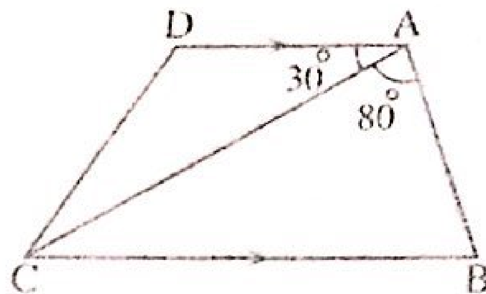


1- In the opposite figure:

$\overline{AD} \parallel \overline{BC}$, m ($\angle$ BAC) = 80° and

m ($\angle$ DAC) = 30°

prove that: BC $>$ AB



2- In the opposite figure:

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that:

AB $>$ CD



First Term-Geometry-Middle (2)

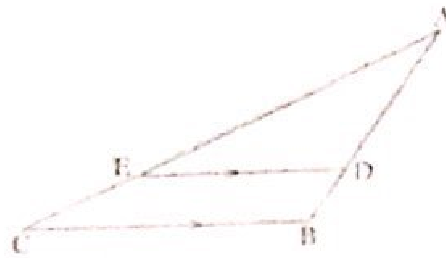
1- In the opposite figure:

ABC is an obtuse-angled triangle at B,

$\overline{DE} \parallel \overline{BC}$

Prove that:

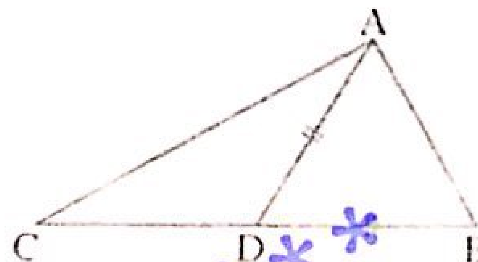
$AE > AD$



2- In the opposite figure:

ABC is a triangle and $D \in \overline{BC}$ where $BD = AD$

Prove that: $BC > AC$



3- In the opposite figure:

$AF = BF = DF$ and $m(\angle FAB) = 50°$

Prove that:

1- $AD > AB$

2- $BC > AC$



Triangle inequality

Generally:

In any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

i.e. In any triangle such as ΔABC,

we get: $AB + BC > AC$

$, BC + CA > AB$

$, CA + AB > CB$



Corollary:

The length of any side in a triangle is greater than the difference between the lengths of the other tw sides and less than their sum.

And you can prove that from the triangle inequality as follows:

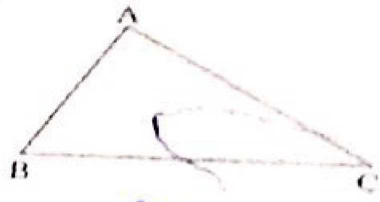
In any triangle ABC:

$AC + AB > BC$ (1)

$, \because AB + BC > AC$ i.e $BC > AC - AB$ (2)

From (1) and (2), we deduce that:

$AC - AB < BC < AC + AB$



Exercise:

1- Is it possible to draw a triangle whose side lengths are as follows? Give reasons:

(a) 3 cm, 4 cm, 9 cm

(b) 5 cm, 7 cm, 8 cm

(c) 10 cm, 6 cm, 4 cm

(d) 13 cm, 8 cm, 6 cm

(e) 5 cm, 3 cm, 4 cm

(f) 9 cm, 9 cm, 19 cm

2- Find the interval to which the length of the third side of the triangle belongs in each of th following triangles if the lengths of the two other sides are:

(a) 6 cm, 9 cm

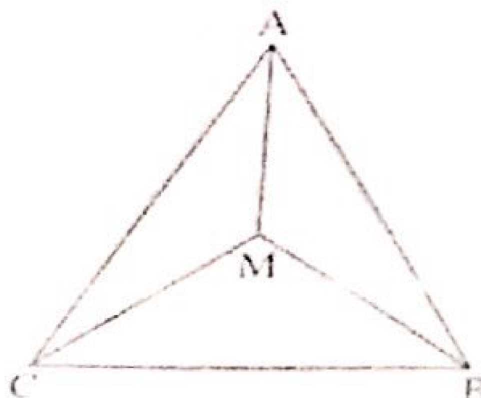
(b) 2.9 cm, 3.2 cm

1- In the opposite figure:

ABC is a triangle in which M is a point inside it.

Prove that:

$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC



General Exercises:

First: Completion questions:

Complete the following:

1- In the isosceles triangle, if AB = AC, m (∠A) = 70°, then AB < ....

2- The longest side in the triangle ABC in which m (∠A) = 105° is ....

3- The shortest side in Δ ABC in which m (∠A) = 40° and m (∠B) = 60° is ....

4- The longest side in Δ XYZ in which m (∠X) = m (∠Y) + m (∠Z) is..........

5- In Δ XYZ, if m (∠X) > m (∠Z), then XY < ....

6- In Δ ABC, if AB > BC, then m (∠A) < ....

7- In Δ ABC, if m (∠A) = 67° and m (∠B) = 33°, then AB > ...... > ......

8- In any triangle, the sum of lengths of any two sides is greater than ....

9- In Δ ABC it will be AB + BC > ....

10- In Δ DEF it will be EF < .... + ...

11- In Δ ABC, if AB < BC < AC, then the smallest angle in measure is ....

12- ABC is an isosceles triangle where AB = 3 cm and BC = 7 cm , then AC = ...

13- An isosceles triangle in which the lengths of two of its sides are 4 cm and 8 cm, then the length o the third side equals ....

Second : Multiple choice questions:

Choose the correct answer from those given:

1- If the lengths of two sides in an isosceles triangle are 2 cm and 5 cm, then the length of the thir side is................

(a) 2 cm (b) 3 cm (c) 5 cm (d) 7 cm

2- If the lengths of two sides in a triangle are 4 cm and 9 cm and it has one axis of symmetry, then th length of the third side is ....

(a) 4 cm (b) 5 cm (c) 9 cm (d) 13 cm

3- Which of the following sets of numbers can be lengths of sides of a triangle?

(a) 2,3,4 (b) 2,3,5 (c) 2,3,6 (d) 2,3,7

4- ABC is a triangle in which m ($\angle C$) = 65° and m ($\angle A$) = 75°, then ....

(a) AB 〉BC (b) AB 〈AC (c) BC〉AB (d) AB = AC

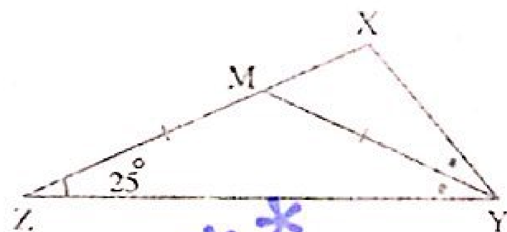
Third : Easy questions:

1- **In the opposite figure:**

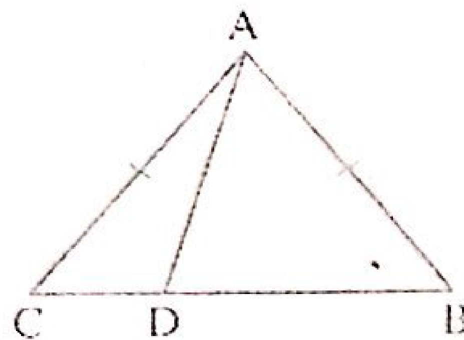
$\overrightarrow{YM}$ bisects $\angle XYZ$, MY = MZ and

M($\angle Z$) = 25°

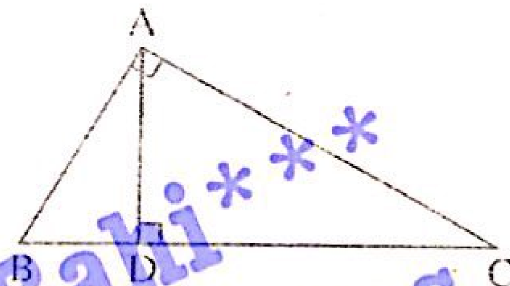
Prove that: YM 〉 XY



1- In the opposite figure:
AB = AC,
D $\in \overline{BC}$
Prove that: AB 〉 AD



2- In the opposite figure:
m ($\angle$ BAC) = 90°,
$\overline{AD} \perp \overline{BC}$ and AC 〉 AB
Prove that: CD 〉 AD



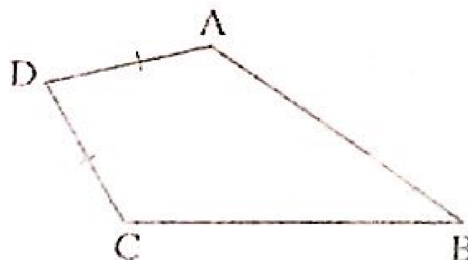
1- In the opposite figure:

ABCD is a quadrilateral in which

AD = CD, m (∠A) > m (∠C)

Prove that:

BC > BA



2- In the opposite figure:

ABCD is a quadrilateral and

$\overline{AC} \cap \overline{BD} = \{M\}$

1- AC + BD > AB + CD

2- AC + BD > AD + BC

3- The perimeter of △BCD < 2(AD + AB + AC)

